Amath Test 12

1. Given that the line $x + y = 6$ meets curve $\frac{1}{x-1} = \frac{3}{y} + \frac{1}{4}$ at A and B. Find the mid-point of AB. [5]

2. Find the value(s) of m such that the expression $mx^2 - 2\sqrt{2}x + m - 1$ will always be positive for all values of x. [4]

3. Prove that $(\tan x + \sec x)^2 = \frac{1+\sin x}{1-\sin x}$ [4]

4. Find the value of p and q for $\frac{2+\sqrt{10}}{2\sqrt{5}+3\sqrt{2}} = p\sqrt{2} + q\sqrt{5}$. [4]

5. Differentiate the following w.r.t. x:

(a) $\frac{1}{3x^2-2}$

(b) $\frac{(x^2-1)}{\sqrt{x}}$

Leave your answer in surd form. [6]

Answer

1. $x + y = 6$

$x = 6 - y$ ----- 1

$\frac{1}{x-1} = \frac{3}{y} + \frac{1}{4}$ ----- 2

Sub 1 into 2,

$\frac{1}{6-y-1} = \frac{3}{y} + \frac{1}{4}$

$\frac{1}{5-y} = \frac{12+y}{4y}$

$4y = (12 + y)(5 - y)$

$4y = 60 - 12y + 5y - y^2$

$y^2 + 11y - 60 = 0$

$(y - 4)(y + 15) = 0$

$y = 4$ or $y = -15$

$x = 2$ $\quad$ $x = 21$

The coordinates of A and B are $(2, 4)$ and $(21, -15)$

The mid-point of AB is $\left(\frac{2+21}{2}, \frac{4+(-15)}{2}\right) = \left(11\frac{1}{2}, -5\frac{1}{2}\right)$

2. $D < 0$

$\left(-2\sqrt{2}\right)^2 - 4(m)(m - 1) < 0$

$8 - 4m^2 + 4m < 0$

$4m^2 - 4m - 8 > 0$

$m^2 - m - 2 > 0$

$m > 2$ or $m < -1$ (rejected)

3. $(\tan x + \sec x)^2 = \left(\frac{\sin x}{\cos x} + \frac{1}{\cos x}\right)^2$

$$= \left(\frac{\sin^2 x}{\cos^2 x} + \frac{2\sin x}{\cos^2 x} + \frac{1}{\cos^2 x}\right)$$

$$= \frac{(\sin x+1)^2}{1-\sin^2 x}$$

$$= \frac{(1+\sin x)^2}{(1+\sin x)(1-\sin x)}$$

$$= \frac{1+\sin x}{1-\sin x} \text{ (proven)}$$

4. $\frac{2+\sqrt{10}}{2\sqrt{5}+3\sqrt{2}} = \frac{(2+\sqrt{10})(2\sqrt{5}-3\sqrt{2})}{2}$

$$= \frac{4\sqrt{5}-6\sqrt{2}+10\sqrt{2}-6\sqrt{5}}{2}$$

$$= 2\sqrt{2} - \sqrt{5}$$

$p = 2, q = -1$

5. (a) $\frac{d}{dx}\frac{1}{3x^2-2} = \frac{d}{dx}\ (3x^2-2)^{-1}$

$$= -1(3x^2-2)^{-2}(6x)$$

$$= -\frac{6x}{(3x^2-2)^2}$$

(b) $\frac{d}{dx}\frac{x^2-1}{\sqrt{x}} = \frac{d}{dx}\ (x^2-1)\left(x^{-\frac{1}{2}}\right)$

$$= 2x\left(x^{-\frac{1}{2}}\right) + (x^2-1)\left(-\frac{1}{2}x^{-\frac{3}{2}}\right)$$

$$= \frac{2x}{\sqrt{x}} - \frac{(x^2-1)}{2x\sqrt{x}}$$

$$= \frac{(3x^2+1)}{2x\sqrt{x}}$$